Permanent vs Transitory Components and Economic Fundamentals*

Anthony Garratt, University of Leicester
Donald Robertson, University of Cambridge
Stephen Wright, Birkbeck College

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Abstract

Any non-stationary series can be decomposed into permanent (or “trend”) and transitory (or “cycle”) components. Typically some atheoretic pre-filtering procedure is applied to extract the permanent component. This paper argues that analysis of the fundamental underlying stationary economic processes should instead be central to this process. We present a new derivation of multivariate Beveridge-Nelson permanent and transitory components, whereby the latter can be derived explicitly as a weighting of observable stationary processes. This allows far clearer economic interpretations. Different assumptions on the fundamental stationary processes result in distinctly different results; but this reflects deep economic uncertainty. We illustrate with two examples: Robertson and Wright’s (2002, 2003) framework for analysing US stock market predictability and risk; and Garratt et al’s (2003, 2004) model of the UK economy.

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1 Introduction

Macroeconomic analysis is largely formulated in terms of stationary processes, yet most economic magnitudes are trending. There is as a result widespread use of a range of de-trending procedures, usually of the “black box” variety, whereby a trend (or permanent component) is extracted by some pre-filtering procedure, usually univariate in nature; with the resulting de-trended series, or transitory component, usually interpreted as a measure of the “cycle”.\(^1\) Such methods typically leave unanswered three key questions: How do we know, from the outset, certain key characteristics that we need to feed into the black box?\(^2\) What are the economic mechanisms that pull a given variable towards its trend? And what, if anything, does the extent of the current estimated deviation from trend tell us about the future of that variable?

This paper argues that analysis of economic fundamentals should instead be central to the process of detrending, and helps to provide important insights (if not necessarily clear-cut answers) to the questions that black box techniques leave unanswered.

This argument is not, we should stress, new in itself. The alternative approach that we advocate is the multivariate Beveridge-Nelson (henceforth B-N) permanent/transitory decomposition\(^3\) that, in turn, provides the basis for a range of alternative multivariate techniques.\(^4\) We do argue however that B-N trends have been unduly neglected, for which there are probably two explanations. The first arises from the (incorrect) perception that B-N trends are “too volatile”.\(^5\) The second is that the process by which B-N trends are usually derived appears, at least, to be even less transparent than that applied in black box univariate techniques.

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1Hodrick & Prescott, 1997; Baxter & King, 1999; Harvey & Trimbur, 2003; Ravn & Uhlig, 2002; Morley, Nelson and Zivot (2003). We prefer to avoid the use of the term “cycle”, which we regard as increasingly a misnomer, since transitory components need not necessarily display periodic movements.

2Typical examples of required prior assumptions might be “smoothness” of the trend; frequency ranges for the “cycle”; or orthogonality restrictions on innovations.

3Beveridge & Nelson, 1981; Stock and Watson 1988a,b; Evans & Reichlin, 1994; Newbold and Arino, 1998

4Since, on reasonable assumptions, the B-N permanent component is a limiting forecast of all possible permanent components derived from a given multivariate representation: for example, the bivariate Blanchard-Quah (1989) decomposition; extended in a larger multivariate framework by King et al (1991) Crowder et al (1999) and Gonzalez and Granger (1995).

5See, for example Massmann and Mitchell 2002; Favero, 2001. Such criticisms are usually both descriptively incorrect in a multivariate context (Evans & Reichlin, op cit) and, we would argue, misplaced, since they prejudge the nature of permanent and transitory components.
Our aim in this paper is to show that, viewed from a new angle, the process that generates B-N permanent and transitory components is not a black box. B-N trends are usually derived from the moving average representation. We show that a very simple alternative derivation, directly from the vector autoregressive representation, has the distinct advantage that transitory components can be related directly to the underlying observable stationary processes that drive the system. The new derivation also helps to illuminate the links with the nature of adjustment processes to disequilibrium.

The advantage of this approach, we argue, is that it should help to focus the mind of applied economists on the key issues involved in the process of detrending. First, we should look for underlying stationary processes, in terms of identifiable economic fundamentals, ideally with a clear basis in theory. Having done this, we need to identify the (necessary) predictive power of such stationary processes for the underlying variables. The transitory components are then simply projections from current values of the underlying stationary processes, and the trends themselves effectively drop out as whatever is left over. The nature of both trends and transitory components must thus depend directly on the nature, and predictive power, of the fundamental stationary processes.

Identifying these processes is of course by no means straightforward. In general, they will reflect deviations - typically due to adjustment costs or other market frictions - from some equilibrium condition. Almost invariably, these equilibrium conditions will link a wider set of variables, hence the process is inherently multivariate in nature. It will usually (but need not always) imply cointegration. There will frequently be competing economic hypotheses about the nature of the equilibrium relationships, both in the long run and (in some cases) in the short run; and very often we cannot avoid a degree of uncertainty about whether the data reject the theory. As a result there may be (and, in the empirical examples we examine, are) significant differences in implied permanent and transitory components, that relate directly to the inclusion or exclusion of certain economic relationships from the system.

This may seem like a disadvantage, but we argue that it is not. Indeed it seems to us to make perfect sense. If we assert that a given series is “above trend” by some amount, we must always at least implicitly be positing some underlying disequilibrium, or set of disequilibria that will, in unconditional expectation, be expected to disappear. Crucially, we are also assuming that a fall (or at least below-average growth) in that series will be an important part of that adjustment process, and that this is to some extent predictable. In our framework, we can directly identify the link between deviations from
trend and the underlying economic disequilibria. But, since there is almost always doubt about the statistical credentials of any process that is assumed to be stationary (whatever its theoretical credentials), we must inevitably end up with different answers, depending on what we assume are the fundamental stationary processes.

We illustrate our analysis with empirical examples that focus on two series, the real value of the US stock market and real UK GDP. We show that the nature of the trends and transitory components is very sensitive to our assumptions about the underlying stationary processes. But, crucially, the source of these differences is clear, because they can be related directly to economic fundamentals with a clear basis in theory. Thus, we argue, theory helps to illuminate the interior of the black box, since the remaining uncertainty about the nature of permanent and transitory components can be related directly to clear economic hypotheses.

The structure of the paper is as follows. Section 2 sets out our new derivation of B-N permanent and transitory components; Section 3 presents our empirical illustrations; Section 4 concludes the paper. An appendix provides details of derivations.

2 Beveridge Nelson Trends in the Cointegrating Vector Autoregressive Representation

2.1 A General Definition

The most general definition of Beveridge-Nelson trends is as limiting forecasts, absent deterministic growth, as the forecast horizon goes to infinity. Thus, for a vector process, \( \mathbf{x}_t \), define the vector of B-N trends, \( \tilde{\mathbf{x}}_t \), by

\[
\tilde{\mathbf{x}}_t = \lim_{h \to \infty} E_t \mathbf{x}_{t+h} - \mathbf{g} h
\]

where \( \mathbf{g} \), the element of deterministic growth, is typically a vector of constants, but may in principle be a deterministic function of \( h \). If \( \Delta \mathbf{x}_t \) can be given a stationary moving average representation of the form

\[
\Delta \mathbf{x}_t = \mathbf{g} + \mathbf{C}(L) \mathbf{\epsilon}_t
\]

then the B-N trends can be expressed as

\[
\Delta \tilde{\mathbf{x}}_t = \mathbf{g} + \mathbf{C}(1) \mathbf{\epsilon}_t
\]

\[\text{For multivariate approaches, see, for example, Newbold and Arino (1998); Evans & Reichlin (1994).}\]
and are thus by definition correlated random walks with drift.

The random walk feature of B-N trends is sometimes represented as a disadvantage, but is a necessary consequence of their forward-looking nature. Thus, suppose we take any arbitrary partitioning of $x_t$ into permanent and transitory components, of the form

$$x_t = x_t^P + x_t^T$$

then, since the transitory components must always satisfy

$$\lim_{h \to \infty} E_t x_{t+h}^T = 0$$

then it must trivially follow that

$$\lim_{h \to \infty} E_t x_{t+h}^P = \tilde{x}_t$$

Thus all possible permanent components must converge in expectation on the B-N trends as the forecast horizon increases. Alternative multivariate techniques that introduce additional assumptions (most commonly orthogonality of innovations, as in Blanchard-Quah (1989); King et al (1991) Crowder et al (1999) and Gonzalez and Granger (1995)) in effect redistribute some additional stationary element between the B-N permanent and transitory components.² We focus in this paper solely on B-N trends in the interests of simplicity and clarity.

2.2 Forecasting from a Cointegrating VAR

When a vector of time series can be given a vector autoregressive representation B-N trends can be derived in a form that is readily interpretable in terms of the underlying stationary processes.³ Assume a cointegrating VAR in $n$ variables, of rank $r$:

$$\Delta x_t = \Psi + \alpha \beta x_{t-1} + \Phi \Delta x_{t-1} + \epsilon_t$$

(4)

or equivalently

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³See Proietti, 1997 for a demonstration of the link between the Gonzalo-Granger and the B-N decompositions.

³The approach here is easily generalisable, in principle, to VARMA processes, as in Arino & Newbold (op cit); although, as they note, identification problems usually rule out such representations on practical grounds. Note that univariate reduced forms from cointegrating VARs will typically be higher order ARIMA processes. We would thus rationalise MA error terms in univariate representations as typically capturing missing cointegrating relations.
\[ \Delta x_t = g + \alpha (\beta' x_{t-1} - \kappa) + \Phi (\Delta x_{t-1} - g) + \varepsilon_t \]  

(5)

where \( x_t \) and \( \Psi \) are \( n \times 1 \) vectors, \( \Phi \) is an \( n \times n \) matrix, \( \alpha \) is an \( n \times r \) matrix, \( \beta' \) is an \( r \times n \) matrix and \( \varepsilon \) is an \( n \times 1 \) vector of error terms. The \( n \times 1 \) vector \( g \) and the \( r \times 1 \) vector \( \kappa \) are the trend growth rates in the variables, and the steady state values of the stationary (typically cointegrating) relationships in levels, respectively and represent the deterministic components of the system. Note that these vectors of constants (\( g \) and \( \kappa \)) can (as shown below) be derived from the intercepts in the estimated VAR as in (4), or can be estimated directly; and hence the data require no pre-filtering (cf. Newbold and Arino, 1998; Rotemberg and Woodford 1996). Higher order VARs can be dealt with by creating new variables for additional lagged differences (which raise \( n \) without raising \( r \)).

Note that \( \beta' x_t \) is typically assumed to pick out cointegrating relationships: stationary combinations of nonstationary series. However \( \beta \) may in principle include columns in which there is only a single non-zero element, thus nesting systems in which one or more series is independently stationary (as, for example, in Blanchard and Quah (1989)-type bivariate representations of output growth and unemployment). In this case the relevant elements of \( \tilde{x}_t \) will be time-invariant. In what follows, however, we use the term “cointegrating relations” as a short-hand for any stationary levels relationships.

The nonstationary system as specified in (4) or (5) can be given equivalent stationary representations. A particularly simple reparameterisation of (4) is as a stationary first-order VAR of the form.

\[ y_t = \begin{bmatrix} \Psi \\ 0 \end{bmatrix} + Ay_{t-1} + v_t \]  

(6)

where

\[ y_t = \begin{pmatrix} \Delta x_t \\ \beta' x_t \end{pmatrix}, \quad v_t = \begin{pmatrix} \varepsilon_t \\ \beta' \varepsilon_t \end{pmatrix}; \quad A = \begin{bmatrix} \Phi & \alpha \\ \beta' \Phi & I_r + \beta' \alpha \end{bmatrix} \]

where \( A \) is \((n + r) \times (n + r)\) and \(| I - A | \neq 0 \). Thus (6) has a unique steady state.

By solving for this steady state, (6) can also be expressed in terms of deviations therefrom (the stationary counterpart to (5)) as the zero mean system

\[ \tilde{y}_t = A \tilde{y}_{t-1} + v_t \]  

(7)

where

\[ \tilde{y}_t = \begin{pmatrix} \Delta x_t - g \\ \beta' x_{t-1} - \kappa \end{pmatrix}; \quad \begin{pmatrix} g \\ \kappa \end{pmatrix} = \begin{bmatrix} \Psi \\ 0 \end{bmatrix} [I - A]^{-1} \]
It is then possible to express $h$ period-ahead expected values of the underlying series as a cumulation of forecasts from the zero mean system:

$$ E_t x_{t+h} = x_t + gh + J \sum_{i=1}^{h} A^i \tilde{y}_t $$

$$ = x_t + gh + B_h \tilde{y}_t $$

$$ = x_t + gh + \alpha_h (\beta^t x_t - \kappa) + \Phi_h (\Delta x_t - g) $$

where $J = \begin{bmatrix} I_n & 0 \end{bmatrix}$ is a selection matrix that picks out $\Delta x_t - g$ from $\tilde{y}_t$, and $B_h = JA[I_{n+p} - A]^{-1}[I_{n+p} - A^{h+1}]$, can be partitioned into two elements, such that $B_h = \begin{bmatrix} \Phi_h & \alpha_h \end{bmatrix}$, where $\Phi_h$ and $\alpha_h$ are of the same dimensions as $\Phi$ and $\alpha$. Each captures the magnitude of expected adjustment, over $h$ periods, to the initial disequilibrium in terms both of cointegrating relations and lagged differences.\(^9\)

Given deterministic growth, as $h$ goes to infinity, conditional forecasts from period $t$ go to infinity, but in deterministically detrended terms, they will go to limiting values given by the “infinite horizon error correction” representation:

$$ \lim_{h \to \infty} (E_t x_{t+h} - gh) = x_t + B_\infty \tilde{y}_t $$

$$ = x_t + \alpha_\infty (\beta^t x_t - \kappa) + \Phi_\infty (\Delta x_t - g) $$

where $B_\infty = \lim_{h \to \infty} B_h = JA[I_{n+p} - A]^{-1}$

In contrast to the 1- or $h$-period error correction representations in (5) and (8) in which the current disequilibria in terms both of cointegrating relations and lagged differences will only be partially eliminated, in the infinite horizon representation all disequilibria must in expectation be fully eliminated. We show below that this property is reflected in a set of restrictions automatically satisfied by the two matrices $\alpha_\infty$ and $\Phi_\infty$.

\(^9\)If the system is generalised such that $\varepsilon_t$, the underlying error process, contains moving average elements of order $p$, there will be an additional term that cumulates the impact, up to horizon $p$, of current and lagged errors. This term will however be constant for $h \geq p$. See Arino and Newbold (1998).
2.3 Beveridge-Nelson Trends as Conditional Cointegrating Equilibrium Values

By inspection the left-hand side of (9) is identical to the left-hand side of (1) that defines multivariate Beveridge-Nelson trends.\(^{10}\) Hence we can interpret B-N trends as conditional cointegrating equilibrium values: the counterfactual values each series would ultimately converge to if all current disequilibria (both in the cointegrating relations and in lagged transitional dynamics) were eliminated. Thus the trend values by definition always satisfy cointegrating equilibrium:

\[
\beta^\prime \tilde{x}_t = \kappa
\]  

(10)

It also follows immediately that the transitory components of the vector \(x_t\) are simply defined by (9) with signs reversed:

\[
x_t - \tilde{x}_t = -\alpha_\infty (\beta^\prime x_t - \kappa) - \Phi_\infty (\Delta x_t - g)
\]  

(11)

A forecast of transitional growth towards ultimate equilibrium at a rate above (or below) steady-state growth thus implies a low (or high) value of the transitory component for any given variable.

A significant difference between the definitions of the B-N permanent and transitory components given here, and the standard formulation, is that our definitions are entirely in terms of current-dated observable economic magnitudes, in contrast to the standard definition in terms of the moving average representation.\(^{11}\) An advantage of our approach is that, as we show in the second of empirical examples, it allows a "cointegrating accounting" approach to analysis of the transitory components. The transitory component of any given variable can be decomposed into the contributions of individual cointegrating disequilibria, and lagged dynamics, respectively. This is particularly helpful when the cointegrating relations have clear economic interpretations.

Note also that, in the special case that \(\Phi_\infty = 0\) there will be an exact static relationship between transitory components and the cointegrating relations. In general this condition will require complicated cross-equation restrictions on the underlying VAR coefficients.\(^{12}\) Since there are only \(r\) cointegrating relations, if the restriction holds, the resulting transitory components will in this case not be linearly independent: a special case of the

\(^{10}\text{For a formal demonstration of the equivalence of our approach with the standard derivation, see Appendix.}\)

\(^{11}\text{Newbold and Arino’s (op cit) and Proietti (op cit) also provide equivalent, but rather more opaque, definitions in terms of observables.}\)

\(^{12}\text{It will however hold automatically in the case of a VAR(1) - a special case we examine in the next sub-section.}\)
“common cycle” approach of Engle & Vahid, 1993; Engle and Kozicki, 1993. Viewed in this light, this version of the common cycle restriction can also be given some theoretical content, since typically there may be little or no theoretical basis for assuming inertia in growth rates, independent of frictions associated with well-identified equilibrium conditions (as captured in cointegrating relations).\textsuperscript{13} Thus the restriction that $\Phi_\infty = 0$ can be interpreted as a restriction that deviations from the set of equilibrium conditions incorporated into the model fully account for observed transitory components of the variables in question. As we shall see below, this restriction appears to be close to holding in both of our empirical examples.\textsuperscript{14}

\section*{2.4 Characteristics of the Infinite Horizon Error Correction Process}

While the infinite horizon error correction representation in (9) that determines the nature of the B-N trends and transitory components has the same structure as the underlying one-period-ahead ECM representation in (5), it turns out to contain distinctly fewer parameters, that are themselves combinations of the underlying VAR parameters.

This can be shown straightforwardly with reference to the definition of the B-N transitory components in (11). Since it is possible that either of the two terms on the right-hand side may in principle be zero at time $t$, the definition of the transitory components must be satisfied with respect to either of the terms alone.

Thus we can consider first the special case where there is an initial disequilibrium in the cointegrating relations ($(\beta'x_t \neq \kappa)$ but growth rates are at their steady-state value ($\Delta x_t = g$) We can now consider the necessary adjustment of the cointegrating relations themselves at an infinite horizon, by pre-multiplying both sides of (11) (with the second term set to zero) by $\bar{\beta}$, thus:

$$\beta' (x_t - \bar{x}_t) = -\beta' \alpha_\infty (\beta' x_t - \kappa)$$

but, since the B-N trends, as noted above, automatically satisfy cointegrating equilibrium, we can substitute from (10), and write

$$\beta' (x_t - \bar{x}_t) = -\beta' \alpha_\infty \beta' (x_t - \bar{x}_t)$$

\textsuperscript{13}An exception to this is sometimes argued to be inflation, although the theoretical basis for inertia in the inflation rate itself is known to be fragile (Kozicki & Tinsley, 2002).

\textsuperscript{14}Note also that, since, as noted below, $\Phi_\infty$ only contains $(n-r) \times n$ free parameters, the number of parameter restrictions required to satisfy this condition is not as large as might at first appear. The number of required restrictions can be shown to be reduced even further if the relationship is allowed to be dynamic, i.e., of the form $x_t - \bar{x}_t = D(L) (\beta' x_t - \kappa)$. 

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implying, by inspection, the adding-up constraint on the elements of $\alpha_\infty$:

$$\beta' \alpha_\infty = -I_r$$

(12)

with the interpretation that, at an infinite horizon, any current disequilibrium must in expectation be fully eliminated. The magnitude of the elements of $\alpha_\infty$ determining the proportion of this necessary adjustment taken up by any given variable. While $\alpha_\infty$ contains $n \times r$ elements, the adding up constraint in (12) implies that these are determined by only $(n - r) \times r$ free parameters.

Similarly, consider the alternative special case where there is an initial equilibrium in the cointegrating relations ($\bar{\beta}' x_t = \kappa$) but growth rates are not at their steady-state values ($\Delta x_t \neq g$). Again consider the necessary adjustment of the cointegrating relations themselves at an infinite horizon, by pre-multiplying both sides of (11) by $\beta'$ (this time with with the first, rather than second term set to zero). Thus:

$$\bar{\beta}' (x_t - \bar{x}_t) = -\beta' \Phi_\infty (\Delta x_t - g)$$

which can only be satisfied if

$$\beta' \Phi_\infty = 0_{r \times n}$$

hence $\Phi_\infty$ is of reduced rank, and only contains $(n - r) \times n$ parameters.

The intuition for this second condition is that, even though, in this special case, the system is assumed initially to be in cointegrating equilibrium, with $\Phi \neq 0$, an initial disequilibrium in growth rates will impart some inertial growth effects that will push the system temporarily away from cointegrating equilibrium again. But (13) says that to the extent that this causes transitory components to be non-zero, this must be consistent with cointegrating equilibrium both at period $t$ and at infinity.

### 2.5 A Simple Example: a Bivariate First-Order Cointegrating VAR

Some further insight can be derived by examining the special case of a bivariate cointegrating VAR(1), with a unit cointegrating vector, of the form:

$$\Delta x_t = \alpha \beta' x_{t-1} + \varepsilon_t$$

(14)

The implied process for the single cointegrating relation is given by

$$\beta' x_t = \frac{\beta' \varepsilon_t}{1 - \theta L}$$

(15)
where $\theta = (1 + \alpha_1 - \alpha_2) < 1$, by assumption.

The simple structure of the underlying VAR implies equally simple expression for the trends and deviations. The two underlying matrices in (9) and hence in (11) are given\(^{15}\) by

\[
\Phi_\infty = 0 \\
\alpha_\infty = -\alpha (\beta'\alpha)^{-1} = \begin{bmatrix} \frac{\alpha_1}{\alpha_2 - \alpha_1} \\ \frac{\alpha_2}{\alpha_2 - \alpha_1} \end{bmatrix} = \begin{bmatrix} -(1 - \xi) \\ \xi \end{bmatrix}
\]

The elements of $\alpha_\infty$ (which captures expected infinite horizon error correction) are in this case just scalar multiples of the corresponding elements of $\alpha$ (which captures expected one-period ahead error correction), given the single eigenvalue. The single parameter $\xi$ that determines both elements of $\alpha_\infty$ ($0 \leq \xi \leq 1$, on the assumption that $\alpha_1 \leq 0$) captures the proportion of eventual adjustment to equilibrium due to changes in $x_{2t}$. The adding-up constraint that any current disequilibrium must be entirely eliminated, as in (12), ensures that the remainder of the adjustment must occur via changes in $x_{1t}$.

The implied process for the transitory components is thus

\[
x_{1t} - \tilde{x}_{1t} = -\alpha_\infty \beta' x_{1t} = -\alpha_\infty \frac{\beta' \varepsilon_t}{(1 - \theta L)}
\]

(16)
a restricted VAR(1), with a single error process that is the innovation to the cointegrating relation, a combination of the underlying VAR errors. Thus the transitory components are perfectly negatively correlated, since they are hit by the same error process, but for a scaling factor, and both have the same autoregressive coefficient as the cointegrating relation itself.\(^{16}\)

The process for the trends themselves can be shown\(^{17}\) to be given by

\[
\Delta \tilde{x}_t = (I - \alpha_\infty \beta') \varepsilon_t \\
= \beta_\perp \alpha'_{\infty\perp} \varepsilon_t = \begin{bmatrix} 1 \\ 1 \end{bmatrix} (\xi \varepsilon_{1t} + (1 - \xi)\varepsilon_{2t})
\]

(17)

where $\beta_\perp$ and $\alpha_{\infty\perp}$ are appropriately normalised orthogonal matrices. Thus the two series have a common B-N trend, the innovation to which (the permanent innovation) is a weighted average of the two VAR innovations, where

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\(^{15}\)Here we exploit the formulation of $B_\infty$, and hence $\alpha_\infty$ in terms of partitioned inverses in the Appendix.

\(^{16}\)This special case of a "common cycle" is also noted by Engle & Vahid (1993).

\(^{17}\)By deriving the implied MA process for $\Delta x_t$, and then either by the conventional B-N approach (see Appendix), or, equivalently, by substitution into (16).
the weights are inversely proportional to their \textit{relative} speed of error correction. The less a series adjusts to disequilibrium, the more closely it will resemble the common trend.

This simple example helps to provide two key insights into the nature of B-N trends, and the link with the error correction representation.

First, there is no necessary result that trends are “smooth” in a multivariate context; equally they need not be “noisy” (a common misperception of B-N trends that largely arises from univariate estimation). The variance of the single permanent innovation in this example can be expressed as

$$\sigma_P^2 = \sigma_1^2 \left[ \xi^2 + (1 - \xi)^2 s^2 + 2\xi (1 - \xi) \rho s \right]$$

where $\rho$ is the correlation coefficient between the two residuals, and $s = \sigma_2/\sigma_1$. The smoothness, or otherwise, of the trend will thus depend both on the relative variances of the two variables, the degree of correlation of individual shocks, and the nature of Granger causality relations (here captured by $\xi$).

Thus, consider the special case of one-way Granger causality (for the sake of argument, from $x_{2t}$ to $x_{1t}$, implying $\xi = 0$). In this case all but the second term will disappear, implying $\sigma_P^2 = \sigma_1^2 s^2 = \sigma_2^2$; and thus the common trend will be relatively “smooth” or “noisy”, compared to $x_{1t}$, depending simply on whether $x_{2t}$ itself is smoother or noisier than $x_{1t}$.

In the case of significant two-way causality, there is a greater presumption that the trend will be relatively “smooth”. In the special case of symmetric mutual causality, for example, $\xi = \frac{1}{2}$, and the variance of the permanent component reduces to:

$$\sigma_P^2 = \sigma_1^2 \left( \frac{1 + s^2 + 2\rho s}{4} \right)$$

Thus if, for example, the two series are equally volatile ($s = 1$), the permanent innovation will have a strictly lower variance than of either of the two underlying innovations.\footnote{Note that this is an identical formula to that for the variance of an equally-weighted portfolio of two assets with imperfectly correlated returns; the relative smoothness of the trend corresponds directly to the well-known gains from portfolio diversification.}

Second, there is no presumption in general that the permanent innovation will be orthogonal to the innovation to the common transitory component. On the other hand, they need not be perfectly correlated (a common misperception that again arises from the nature of univariate B-N trends). The covariance of the permanent and transitory innovations will be given in the
above example by

$$\sigma_{PT} = \sigma_1^2 \left[ \xi - (1 - \xi)s^2 + 2 \left( \frac{1}{2} - \xi \right) \rho_8 \right]$$

which will in general be of indeterminate sign.\textsuperscript{19}

We have of course abstracted, in this simplified analytical example, from the impact of more complex dynamics.\textsuperscript{20} Our empirical examples suggest, however, that even when the underlying VAR is higher order these play a relatively limited role: i.e., in both cases we look at the particular common cycle restriction seems to be close to holding, such that the transitory components can be mapped nearly precisely onto the cointegrating relations.

3 Empirical Illustrations

3.1 Valuation Indicators and the US Stock Market

3.1.1 The General Framework

In our first example, we draw on a sequence of papers by Robertson and Wright that analyse the role of two alternative valuation indicators in predicting aggregate stock returns. The research draws on a new annual dataset for the US nonfinancial corporate sector, covering the period 1900-2002, described in detail in Wright (2001). The key innovations of these papers are: i) the use of a measure of the dividend yield defined in terms of total cashflow to shareholders (including non-dividend cashflows such as repurchases or cash-financed M&A) in place of the more commonly used narrow dividend yield (Robertson & Wright, 2003); ii) in analysing the role of Tobin’s $q$ in predicting returns, Robertson and Wright (2002a;b).

\textsuperscript{19}It will be equal to zero in the case of symmetric two-way causality, and equal variance of the underlying innovations ($\xi = \frac{1}{2}; s = 1$); but this is a highly special case.

\textsuperscript{20}If we supplement the bivariate example above such that is of the same form as (5), and is thus a VAR(2) in levels, the transitory components can be expressed, using the reduced rank feature of $\Phi_\infty$ as:

$$\mathbf{x}_t - \hat{\mathbf{x}}_t = -\mathbf{x}_\infty (\beta' \mathbf{x}_t - \kappa) - \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Delta z_t$$

where the scalar process $z_t = \psi'(\mathbf{x}_t - g)$ is a weighting of the two disequilibrium growth rates. Thus the two transitory components share an identical second element, even without any supplementary restrictions on $\Phi_\infty$. The relative magnitudes of the elements of $\psi$ again depend on the nature of Granger causality relations. In the special case of one-way causality from $x_{2t}$ to $x_{1t}$, for example, the first element of $\psi$ will be zero, and the second element will be determined solely by the univariate properties of $x_{2t}$. 

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In terms of the modelling framework set out in the previous section, we set \( x_t = [v_t \ d_t \ k_t \ l_t]^T \), where the underlying real processes (all expressed in logs), are stock market value \((v_t)\); (true cashflow) dividends \((d_t)\); the capital stock \((k_t)\); and net corporate liabilities \((l_t)\). All are nonstationary, and are assumed to be \( I(1) \).

### 3.1.2 Valuation Indicators and Implied Cointegrating Relations

The two valuation indicators imply two candidate cointegrating relations in the system set out above. The cashflow dividend yield is defined by

\[
c_t = d_t - v_t = [-1 \ 1 \ 0 \ 0] x_t = \beta_1' x_t
\]

while Tobin’s \( q \) can be expressed, up to a linear approximation, as

\[
q_t = (1 - \lambda)(v_t - k_t) + \lambda(l_t - k_t) = [1 - \lambda \ 0 \ -1 \ \lambda] x_t = \beta_2' x_t
\]

where \( \lambda \) is approximately equal to the historic mean leverage ratio.

The stock return itself is defined using a close relative of the much-used Campbell-Shiller (1988) approximation for the log return, by:

\[
\log(1 + R_t) = \delta v_t + (1 - \delta)(d_t - v_t)
\]

where \( \delta \) is a small fraction approximately equal to the historic mean cashflow yield (around 0.04). Thus real return uncertainty is dominated by uncertainty in the real value of the stock market: in this paper, for simplicity, we concentrate on this element alone.

On both theoretical and empirical grounds, it is reasonable to assume that both the valuation indicators, and real returns, will be stationary processes (indeed the latter assumption requires that \( c_t \) be stationary, if \( v_t \sim I(1) \)). If this is the case, there will be 2 cointegrating relations in a system of four variables, i.e.: \( \beta = [\beta_1' \ \beta_2' ] \).

Robertson & Wright (2002a) note that, while \( q_t \) appears stationary on standard tests, a representation in which \( r = 1 \), with the cashflow yield as the sole cointegrating relation \((\beta = \beta_1)\) is also statistically acceptable. Robertson and Wright (2002b) show, however, that the inclusion of \( q_t \) as a cointegrating relation in the system markedly reduces long-horizon return uncertainty. Here, we show that this property can be directly related to the nature of the associated B-N trends.

### 3.1.3 Permanent vs Transitory Movements and Economic Fundamentals in the US Stock Market

Figures 1 and 2 illustrate the impact of imposing different rank and structures on \( \beta \). In order to bring out the impact of any estimated disequilibrium in
the recent past, the charts project actual and trend series to 2030, from the terminal observation of end-2002.

Figure 1 shows actual and trend values of the four elements in the system, when the cashflow dividend yield is the single cointegrating relation. To a reasonable approximation, this results in a system in which two of the series \((v_t \text{ and } d_t)\) effectively form a distinct bivariate system, and hence share a common stochastic trend, while the remaining two series \((k_t \text{ and } l_t)\) are very close to being univariate random walks.\(^{21}\)

The nature of the implied common trend in the first two series can be readily explained in relation to the analysis of the bivariate example in the previous section. Corporate cashflow, the denominator of the cashflow dividend yield, is, as the chart makes clear, a very noisy series (in contrast to conventional measures of dividends): indeed its innovations are of similar variance to those of the stock market itself. Empirical tests suggest two-way Granger causality (consistent with predictability of returns, since, as noted above \(r_t \approx \Delta v_t\)), but with the predictive power of the cashflow yield for stock market value being relatively weak (though statistically significant). As a result, in line with the analysis of Section 2.5, the common trend is somewhat smoother than both series, but resembles stock market value rather more closely than it resembles corporate cashflow.

Nonetheless there were important points in history when stock market value was significantly above or below its trend (corresponding to a low or high cashflow yield): most notably in the late 1920s (when the growth in market value became significantly divorced from that of corporate cashflow), and in the early 1970s (when cashflow weakened in advance of the stock market).

In contrast to these events, during the great stock market boom of the 1990s, and the sharp falls in the stock market at the turn of the millennium, neither series deviated much from its B-N trend, since both grew broadly in line with each other (and both at a quite exceptional pace) through the boom; both then fell back by similar amounts. Since the single cointegrating relation, the cashflow yield, at no point deviated very significantly from its mean, the transitory component of both series showed very little movement: thus both the rise and fall in the market must be interpreted, in this framework, as permanent shocks.

Figure 2 shows that this is far from being the case when Tobin’s \(q\) is

\(^{21}\text{Note that in both } r = 1 \text{ and } r = 2 \text{ cases, a common deterministic growth rate for all four series was imposed (and easily accepted) in estimation, ruling out deterministic bubbles in any of the underlying ratios. Thus in steady state all four series grow at the same rate, as is evident in the projections to 2030 (note that all four panels have the same range in logarithmic terms).}\)
included as a second cointegrating relation. The nature of the trends is distinctly different in a number of important respects.

With two cointegrating relations, there can be only two independent stochastic trends, rather than three, as in the \( r = 1 \) case. Figure 2 reveals that one of these is essentially given by the capital stock itself: the fact that it deviates only to a very modest extent from its underlying trend is consistent with the well-known result that \( q \) has relatively weak predictive power for investment. The trend in \( v_t \) and \( d_t \) is now effectively a combination of the trend in the \( r = 1 \) case, and the capital stock. It is as a result distinctly smoother throughout the sample period. The difference from the \( r = 1 \) case is however most marked in the 1990s. While the (virtually identical) B-N trends in \( v_t \) and \( d_t \) are pulled up by the strong growth in the two series themselves during that period, this growth is significantly muted by the steadier growth of the capital stock. The counterpart to this was a very sharp rise in \( q_t \), to historically unprecedented levels.

As a result, both series were, by the end of the 1990s, well above their trends, and then fell sharply back towards them (though both remained above trend as of end-2002). Thus, in stark contrast to the \( r = 1 \) case, this representation implies that the rise and fall of the market was almost entirely a movement in transitory, rather than permanent components.\(^{22}\) It also implies that the adjustment of the first two years of the millennium was far from complete: the implied projections beyond then are of very slow growth in both stock market value and corporate cashflow until \( q \) is brought back to its mean.

The very marked contrast between these two interpretations of the data (that are, it should be noted, econometrically virtually indistinguishable) is not, it should be stressed, a black box phenomenon, but can be traced back to assumptions about the fundamental stationary relationships, and their predictive power. If Tobin’s \( q \) is a stationary process, and historic adjustment to disequilibria are a good guide to future adjustments, the logical implication must be that the 1990s stock market boom was a transitory phenomenon. If \( q_t \) is not a stationary process, the rise and fall of the market must be interpreted as a sequence of permanent shocks.

But the implications do not stop there. If the latter interpretation is correct, and the only stationary relationship is the cashflow dividend yield, then the resulting common trend in corporate cashflow and the stock market is very much more volatile than it is if Tobin’s \( q \) is also stationary. Over long horizons, the key element in uncertainty in any series is the uncertainty in

\(^{22}\)A finding very similar to that of Lettau and Ludvigsen (2001), on the basis of a cointegrating relation between consumption, wealth and labour income.
its permanent component. The variance of the stationary transitory component goes to a finite limit as the forecast horizon lengthens, but that of the permanent component rises without limit.\textsuperscript{23} Thus, the more volatile is the permanent component, the greater is long-horizon uncertainty in both corporate cashflow and the value of the stock market. Uncertainty in the latter is, as noted above, the key determinant of long-horizon return uncertainty. Thus, as Robertson & Wright (2002) note, if Tobin’s $q$ is a cointegrating relation, this is bad news for expected returns in the immediate future, but it is good news for return variance over longer horizons.

### 3.2 A Small Model of the UK Economy

#### 3.2.1 The General Framework

In the second of our examples we focus on the transitory components rather than the trends. Specifically we examine the transitory movements in UK GDP, using the model of Garratt \textit{et al.} (2003; 2004), where we consider the following set of quarterly variables for the UK economy:

$$x_t = (p_t^0, e_t, r_t^*, r_t, \Delta p_t, y_t, p_t - p_t^*, h_t - y_t, y_t^*, t)^T.$$

$p_t^0$ is the logarithm of oil prices, $e_t$ is the logarithm of the nominal exchange rate (defined as the domestic price of a unit of the foreign currency, so that a depreciation of the home currency increases $e_t$), $r_t^*$ is the foreign short term nominal interest rate, $r_t$ is the domestic short term nominal interest rate, $p_t$ is the logarithm of domestic prices, $y_t$ is the logarithm of real per capita domestic output, $p_t^*$ is the logarithm of foreign prices, $y_t^*$ is the logarithm of real per capita foreign output, $h_t$ is the logarithm of the real per capita money stock and $t$ is a deterministic time trend.

Garratt \textit{et al.} work on the assumption that all variables are I(1) and estimate a cointegrating VAR(2) model in which they examine the impact of imposing cointegrating relations based on theory. We examine the impact of these restrictions in Section 3.2.3 below. First, however, for the purposes of illustration, we examine the consequences of simply deriving multivariate B-N trends of the black box variety.

#### 3.2.2 Pitfalls of Atheoretic Multivariate De-Trending

The previous empirical example demonstrated that there can be quite distinct difference in implied trends and transitory components, depending on

\textsuperscript{23}Absent parameter uncertainty, forecast variance increases linearly with the forecast horizon.
whether one or two cointegrating relations were imposed - even when the implied restrictions could easily be imposed on the data. In a larger model (eight endogenous variables and one exogenous variable) the problem is distinctly more complex since uncertainty regarding the correct multivariate empirical representation of the data is extremely high.

A particular problem is the uncertainty both about $r$, the rank of $\beta$, and for any given rank, the nature of the cointegrating relationships. Alternative approaches to determining the rank can yield quite different values, especially in relatively large models. Given this uncertainty we briefly illustrate the impact of a range of assumptions regarding the rank of $\beta$ on the properties of the transitory components. The nature of the exercise is deliberately atheoretical in the sense that it does not impose any restrictions on the matrix $\beta$ except those required for exact identification. Thus we focus only on the impact of rank uncertainty.

We analyse a range of rank restrictions, $r = 0$ through to 7, where $\beta$ is exactly identified. Note that the $r = 0$ case where no long run relationship exists provides us with a useful benchmark with which to compare the effect of imposing long run relationships. It also produces results very similar to the univariate Beveridge-Nelson decomposition.

Figure 3 plots the transitory components in GDP for all eight exact identified cases for the period 1965q1-1999q4 (we denote the exactly identified models of ranks 0 through to 7 as $Ex0, Ex1, \ldots, Ex7$). The chart makes clear that an atheoretic approach to the multivariate B-N decomposition provides little or no guidance on the nature of transitory movements in UK GDP. The variance and even signs of the resulting transitory components differ so markedly, depending on the rank we impose, that in the absence of any further information, the “black box” approach essentially provides no useful insights at all.

### 3.2.3 Cointegrating relations based on theory

Given that the atheoretic approach produces such varied and inconclusive results, we now turn to the impact of imposing cointegrating relationships

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24Different econometric tests of the cointegrating rank of $x_t$ yield a wide range of different values. Garratt et al (2003) conclude that the weight of the evidence is that $r = 5$; but there is sufficient uncertainty surrounding its exact value that it is of interest to see the implications of assuming different values.

25There is some indication that the standard deviation of the transitory component increases with $r$ (and, by implication, innovations to trend become lower variance, consistent with Evans & Reichlin, op cit). A possible interpretation of this is that, as rank rises, the implied trend values consistent with cointegrating equilibrium are a very long way from current values, but with increasingly slower adjustment speeds towards equilibrium.
based on theory. Garratt *et al.* (2003) examine the impact of imposing five long-run relationships which were argued to be important to a small open economy like the UK. In brief, we note here that the five fundamental relationships that are assumed to result in stationary processes (and hence provide the structure for $\beta$) are:

(i) Purchasing Power Parity (PPP), implying $p_t - p_t^* - e_t \sim I(0)$;
(ii) Interest Rate Parity (IRP), implying $r_t - r_t^* - \Delta e_t \sim I(0)$;
(iii) Convergence (CONV), implying $y_t - y_t^* \sim I(0)$;
(iv) Stable Real Money Demand (RMB) implying $h_t - y_t + \zeta_1 r_t + \zeta_2 t \sim I(0)$;
(v) Fisher Interest Parity (FIP) implying $r_t - \Delta p_t \sim I(0)$,$^{26}$

In contrast to the atheoretical exercise the attempt to relate the long run to explicit theory implies the presence of over-identifying restrictions. Figure 4 plots the transitory component in UK GDP derived from the benchmark overidentified model ($Ov5$) alongside the atheoretical exactly identified ($Ex5$).

From the plot we observe only a limited degree of co-movement between the two series (with a correlation coefficient of only 0.36). The size of the deviations also differs significantly (the standard deviation for $Ov5$ and $Ex5$ are 2% and 3% respectively) and it is clear that imposing the long run restrictions has implications for output deviations over and above just imposing the required rank restriction.$^{27}$

### 3.2.4 The Transitory Component in UK GDP and Economic Fundamentals

The great advantage of the benchmark restricted model is that it is possible to identify the link between the assumed underlying fundamental processes (which have clear economic interpretations) and the resulting transitory components. Equation (11) showed that the transitory components can be broken down into the contribution of the cointegrating relations, and of inertial effects in lagged differences. Figure 5 shows that movements in the transitory component in UK GDP are overwhelmingly dominated by movements in cointegrating relations: the role of short-run dynamics is very limited.$^{28}$

$^{26}$The structure of the model, with both $p_t - p_t^*$ and $e_t$ assumed to be $I(1)$, implies that domestic and overseas inflation rates and interest rates can differ by at most a constant in steady state.

$^{27}$The statistical counterpart to the significant difference between the two series is a rejection of the implied restrictions on a conventional likelihood ratio test (see Garratt *et al.*, 2003 for a discussion, and a comparison with bootstrapped test statistics, which do not reject the restrictions).

$^{28}$Suggesting the presence of common cycles. However, it should be borne in mind that we are only examining the transitory component in one of the eight variables in the model.
Figure 6 provides a more detailed decomposition of the transitory component by breaking it down into the contributions of individual cointegrating relations. Probably the most striking feature of this chart is that there are large, but frequently offsetting contributions from the two cointegrating relations that include output itself, CONV and RMB. To accentuate this feature Figure 7 aggregates the impact of these two relationships: this shows that in most periods these two relationships account for most of the variations in the transitory component. The only other relationship that plays a significant role is the PPP relationship. Figure 8 shows the impact of excluding each of these three key relationships in turn. These are shown alongside the transitory component from the \( r = 5 \) benchmark model. The chart shows that excluding either CONV or RMB has particularly marked effects.\(^{29}\)

3.2.5 Interpretation

As in our first example, the differences in transitory components discussed above can be related to interpretable economic hypotheses. Our results suggest that two relationships: convergence and the real money demand relation, are key; with purchasing power parity being the only other relationship of any significance for output.

The relative lack of importance of the other two relationships, international interest parity and Fisher interest parity, for output is consistent with standard small open economy assumptions. These would suggest, first that both real and nominal interest relations are determined by world markets, and second, as a result, that the role of domestic real interest rate movements in output fluctuations is quite small (as compared to the role of the real exchange rate).\(^{30}\)

The important role of the convergence relationship is striking, since it suggests a strong link between two strands of macroeconomics - growth and fluctuations. Thus, Figures 6 and 8 show that the convergence relationship made a very large negative contribution to the transitory component in output during the 1970s and early 1980s. The implication was that, after the relatively slow growth of the UK in the 1960s and 1970s, compared to its competitors, the convergence relationship was, by the mid-1970s, expected to make a major contribution to UK growth in the 1980s and 1990s. This catching-up phase did indeed subsequently emerge. While this prediction is

\(^{29}\)We have also carried out a systematic comparison of all possible combinations of the five candidate cointegrating relations, that reinforces this conclusion. Details are available on request from the authors.

\(^{30}\)Note that these relationships are of considerably greater importance to the transitory components of other variables in the system.
consistent with the convergence literature, its impact on shorter-term output fluctuations has been largely ignored. The assumption that relative output levels is a stationary process (which is fundamental to the convergence literature), and, crucially, has such important predictive power for domestic output, puts a very different interpretation on the transitory component to the standard “cycle”, since the associated stationary processes are much more long-term in nature than typically assumed.

At the same time the model also suggests a quite significant “monetarist” interpretation, given the important role of the real money demand relationship. When both of the two key relationships are included, there are, as noted above, significantly offsetting impacts on the transitory component in output. Thus, the convergence relationship would, in isolation, have implied a significantly negative transitory component during the mid-1970s and early 1980s. But this was to a considerable extent offset by a positive contribution from the real money demand relationship. When the money demand relationship is excluded, Figure 8 shows that movements in the transitory component in output become much smaller. This implies, in turn, that output movements are much more dominated by permanent shocks, consistent with the real nature of the remaining cointegrating relationships. On the other hand, when the convergence relationship is omitted, and the transitory component is dominated by disequilibrium in the real money demand relationship, there are much larger and more persistent swings.

If the money demand relationship is taken at face value, this implies a quite significant role for money (or nominal) shocks in fluctuations in UK output. However, there may be some reason for at least a degree of scepticism about this conclusion. The nature of the real money demand relationship is, in one crucial respect, very distinct, in econometric terms, from that of the other cointegrating relations, since it is the only one that contains any estimated cointegrating parameters. All four of the remaining long-run relationships imply cointegrating vectors of the “(1,-1)” variety, where the coefficients arise straightforwardly from very basic theory. Thus, three of the equilibrium relationships: convergence; international interest parity; and purchasing power parity, are effectively no arbitrage conditions. The fourth, Fisher interest parity, requires the absence of nominal illusion, coupled with some stability of intertemporal preference parameters. The nature of the associated hypotheses leaves no scope for the data to provide estimated cointegrating parameters (except for constant terms) under the null.

In contrast, the nature of the hypothesis associated with the real money demand relationship is distinctly less constrained. Theory only predicts the sign, but not the magnitude, of the response of money demand to the nominal interest rate; nor does it rule out a role for a deterministic time trend. As a
result, in this relationship, two parameters are chosen, not, as in the case of
the other relationships, on the basis of theoretical priors, but on the basis of
the resulting ability of the model to fit the data. There is therefore a potential
data-mining critique of this relationship, that does not apply to the other
four. One interpretation of the offsetting nature of the contributions from
the convergence relationship and money demand to the transitory component
in output is that the free coefficients in the latter effectively allow it to “pick
up the slack”, in predictive terms, from the former. Given this caveat, we
would hesitate before concluding that disequilibrium in money demand has
clearly had as important a role in UK output fluctuations as the benchmark
model would suggest.\textsuperscript{31}

The only other factor that plays a significant role in transitory output
fluctuations is the PPP relationship. At certain points, deviations of the
real exchange rate from its estimated (constant) equilibrium value imply a
contribution to output fluctuations that is non-trivial. Thus in the early
1980s the strength of sterling (which the PPP relationship implies must have
been a transitory phenomenon) implied that output was depressed below its
long-run trend value; by a maximum amount of around 2\% in 1981 when
sterling was at its strongest. Perhaps more surprising is the implied role of
the real exchange rate in more recent output fluctuations. During the ERM
period of the early 1990s, sterling was widely regarded as over-valued; the
sharp devaluation at the end of 1992 was at the time regarded as restoring
a more sustainable real value of sterling. Figure 6 provides a distinctly
different interpretation. The contribution of the PPP relationship to the
transitory component of output at the start of the 1990s was essentially zero,
for the simple reason that, on the basis of model estimates, sterling was at
its equilibrium in terms of PPP. The subsequent devaluation shows in the
chart as a move \textit{away} from equilibrium, with the PPP relationship making a
distinctly positive contribution to the transitory component of output during
the mid-1990s. This contribution must, of its nature, be interpreted as
transitory; and indeed, by the end of the 1990s the model suggests that it
had evaporated to virtually zero, with sterling, by implication, again at its
equilibrium in terms of PPP.

At the very end of the sample period, in 1999, the benchmark model
suggests that output was nearly 2\% above trend: largely accounted for by the
convergence relationship (UK output having by then grown distinctly faster
than overseas output for a number of years), with some negative offsetting

\textsuperscript{31}One area that would repay further investigation is whether the apparent relative im-
portance of this relationship would persist if econometric techniques were employed that
were less prone to a data-mining critique: for example, recursive estimation. It would also
be interesting to discover if this feature were robust to alternative measures of money.
impact from real money demand. Figure 8 shows that, as on many other occasions, this conclusion is however quite sensitive to the assumptions on the underlying fundamental stationary processes. The chart shows that when the convergence relationship is omitted from the model, output is estimated to have been below, rather than above trend at this time (a feature shared with a number of the atheoretical transitory components plotted in Figure 3). But, as we have argued above, this degree of ambiguity is inevitable. If we assume that convergence results in a stationary process for relative output levels, then the nature of output fluctuations must of necessity be different from a world in which we assumed it was not. The nature of transitory fluctuations cannot be separated from the nature of the underlying fundamental stationary processes that drive the economy.

4 Conclusions

In this paper we present a new derivation of multivariate Beveridge-Nelson trends from the cointegrating vector autoregressive representation, that allows us to relate movements in permanent (or trend) and transitory components directly to the underlying stationary processes. We interpret B-N trends as conditional cointegrating equilibrium values, and show how the nature of the permanent and transitory components can be related to the nature of the error correction process, at both finite and infinite horizons.

We have argued that the role of theory is crucial in suggesting equilibrium relationships; but equally econometric evidence is crucial in revealing the nature of the adjustment towards equilibrium. Neither theory nor econometric evidence can eliminate uncertainty about the nature of permanent and transitory components; but, as our two empirical examples have demonstrated, both are crucial in clarifying the nature of this uncertainty. As such, we have argued that an approach to the derivation of permanent and transitory components that is based on the analysis of fundamental stationary processes has distinct advantages over the atheoretical (and typically univariate) detrending processes that are still very widely applied. We do not claim that it can provide clear-cut answers to the questions that such atheoretical approaches leave entirely unanswered; but we do claim that it at least provides a coherent framework in which those questions can be investigated.
Appendix

A Equivalence of Trend Definition in (10) to the Standard Multivariate Beveridge-Nelson Definition

For convenience, we first write (5) in zero mean form, as

$$\Delta \tilde{x}_t = \alpha \beta' \tilde{x}_{t-1} + \Phi \Delta \tilde{x}_{t-1} + \varepsilon_t$$

where

$$\tilde{x}_t = x_t - gt - \gamma,$$

and $\gamma$ is any vector in the space defined by $\beta' \gamma = \kappa$. We can reformulate this, following Hansen and Johansen (1998), as a first-order system

$$\Delta x_t^* = \alpha^* \beta^* x_{t-1}^* + \varepsilon_t^*$$

where

$$x_t^* = \left( \tilde{x}_t', \tilde{x}_{t-1}' \right)' \quad \varepsilon_t^* = \left( \varepsilon_t', 0 \right)'$$

$$\alpha^* = \begin{pmatrix} \alpha & \Phi \\ 0 & I_n \end{pmatrix} \quad \beta^* = \begin{pmatrix} \beta & I_n \\ 0 & -I_n \end{pmatrix}$$

Note that in this representation, $\beta^*$ constructs both cointegrating relations and lagged difference terms. Having simplified to first-order form, we can redefine (7) from the main text in the simpler form in terms of the $2n + r$ vector $y_t^*$:

$$y_t^* = A^* y_{t-1}^* + v_t^*$$

$$y_t^* = \begin{pmatrix} \Delta x_t^* \\ \beta' x_t^* \end{pmatrix} \quad v_t^* = \begin{pmatrix} \varepsilon_t^* \\ \beta' \varepsilon_t^* \end{pmatrix}$$

$$A^* = \begin{bmatrix} 0 & \alpha^* \\ 0 & I + \beta' \alpha^* \end{bmatrix}$$

We can then derive trend values in parallel fashion to the main text as

$$\hat{x}_t^* = x_t^* + J^* A^* (I - A^*)^{-1} y_t^*$$

$$x_t^* + B_{\infty}^* y_t^*$$

---

32 Higher order lagged difference terms can also be incorporated, as noted in the main text, by redefinition of $\Phi$. 
where $\mathbf{J}^* = [ \mathbf{I}_{2n} \ 0 ]$.

Given the simpler definition of $\mathbf{A}^*$, compared to $\mathbf{A}$ in the main text, we can use the partitioned inverse formula to get

$$(\mathbf{I} - \mathbf{A}^*)^{-1} = \begin{pmatrix} \mathbf{I} & -\alpha^*(\beta''\alpha^*)^{-1} \\ 0 & -(\beta''\alpha^*)^{-1} \end{pmatrix}$$

So

$$\hat{x}^*_t = \mathbf{x}^*_t + \mathbf{J}^* \begin{pmatrix} 0 & \alpha^* \\ 0 & \mathbf{I} + \beta''\alpha^* \end{pmatrix} \begin{pmatrix} \mathbf{I} & -\alpha^*(\beta''\alpha^*)^{-1} \\ 0 & -(\beta''\alpha^*)^{-1} \end{pmatrix} \mathbf{y}^*_t$$

$$= (\mathbf{I} - \alpha^*(\beta''\alpha^*)^{-1}\beta'') \mathbf{x}^*_t$$ (23)

To obtain the usual Beveridge-Nelson decomposition we first need to obtain the moving average representation for $\Delta \mathbf{x}^*_t$. Using (20), we can substitute for $\bar{\mathbf{x}}^*_t - 1$ in (20) and obtain

$$\Delta \mathbf{x}^*_t = \mathbf{C}(L) \mathbf{e}^*_t$$ (24)

where $\mathbf{C}(L) = \mathbf{I} + \alpha^* (\mathbf{I} - (\mathbf{I} + \beta''\alpha^*) L)^{-1} \beta'' L$

The conventional definition of Beveridge-Nelson trends is:

$$\hat{x}^*_t^{BN} = \mathbf{C}(1) \sum \mathbf{e}^*_t$$ (25)

where $\mathbf{C}(1) = \left( \mathbf{I} - \alpha^* (\beta''\alpha^*)^{-1}\beta'' \right)$, and, using this expression, we can decompose $\mathbf{x}^*_t$ into its permanent and transitory components as

$$\mathbf{x}^*_t = \mathbf{C}(1) \sum \mathbf{e}^*_t + \mathbf{D}(L) \mathbf{e}^*_t$$ (26)

where

$$\mathbf{D}(L) = (1 - L)^{-1} (\mathbf{C}(L) - \mathbf{C}(1))$$

$$= \left( \alpha^* (\mathbf{I} - (\mathbf{I} + \beta''\alpha^*) L)^{-1} \beta'' L + \alpha^* (\beta''\alpha^*)^{-1}\beta'' \right) (1 - L)^{-1}$$

At this stage we can observe that, on our definition, as in (23)

$$\hat{x}^*_t = \mathbf{C}(1) \mathbf{x}^*_t$$

$$= \mathbf{C}(1)^2 \sum \mathbf{e}^*_t + \mathbf{C}(1) \mathbf{D}(L) \mathbf{e}^*_t$$

thus, equivalence of the two trend definitions reduces to two conditions:
The first of these, that $C(1)$ is an idempotent matrix, can be verified straightforwardly. The second can be shown to hold by writing

\[
C(1)D(L) = 0
\]  

since $(I - \alpha^*(\beta'\alpha^*)^{-1}\beta')\alpha^* = 0$ by taking the $\alpha^*$ into the brackets. Thus the two trend definitions are identical.
Figure 1: $r = 1$ Model with Cashflow Dividend Yield.
Figure 2: $r = 2$ Model with Cashflow Dividend and Tobins $q$. 
Figure 3: Transitory Components in UK GDP from Exactly Identified Models of Different Rank

Figure 4: Transitory Component of UK GDP with (Ov5) and without (Ex5) Long-Run Theory.
Figure 5: Decomposition of Benchmark Transitory Output Component into Long and Short Run Deviations.

Figure 6: Contribution of Cointegrating Relations to the Transitory Component in GDP
Figure 7: The Combined Impact of Cointegrating Relations including Output.

Figure 8: Impact on the Transitory Component of GDP of Excluding Cointegrating Relations
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