Problem Set 1
(Due: February 17)

1. The scores of the final exam of 20 students in class X is {88,91,88,85,82,82,94,76, 71,85,86,69,72,68,67,66,61,69,63,54}.

   (a) Calculate the mean, median, mode, variance, standard deviation, range, and the inter-quartile range. Also calculate the z-score of the student with score 82.

   (b) Make the stem-and-leaf plot. Also draw a histogram with a class width of 10 and with the lowest value being 50. That is, 50-59 is a class, and the next class is 60-69, and so on. Make a table of cumulative relative frequency based on the histogram and draw a corresponding ogive.

   (c) Calculate the weighted mean using the histogram only (i.e., assuming that you only know the histogram but not exact scores). Is this weighted mean using grouped data the same as the mean calculated using raw data?

2. Continued from the above, suppose you also know the scores of the midterm. They are {107,103,107,109,103,91,108,110,95,94,105,98,101,96,95,95,87,85,72,69}. The sequence of the midterm and final scores are made according the same roster. Use Excel to make a scatter diagram and calculate the covariance and correlation coefficient. Please well label your Excel spreadsheet and submit a printout of your Excel spreadsheet and graph.

3. $A \equiv \{x | 1 \leq x \leq 20 \ \& \ x \ is \ an \ integer\}$?
   $B \equiv \{x | 1 \leq x \leq 20 \ \& \ x \ is \ an \ even \ number\}$?
   $C \equiv \{x | 1 \leq x \leq 20 \ \& \ x \ is \ an \ odd \ number\}$?
   $D \equiv \{x | 1 \leq x \leq 10 \ \& \ x \ is \ an \ integer\}$?
   $E \equiv \{x | 10 \leq x \leq 20 \ \& \ x \ is \ an \ integer\}$?
   $F \equiv \{x | 1 \leq x \leq 20 \ \& \ x \ is \ a \ multiple \ of \ 3\}$?
   $G \equiv \{x | 1 \leq x \leq 20 \ \& \ x \ is \ a \ multiple \ of \ 5\}$?
   Suppose A is the universal set.

   (a) Find B′(B′ for the complement of B)?

   (b) Find $B \bigcap D$ and $B \bigcup D$.

(c) Find $(B \bigcap F)'$ and $B' \bigcup F'$ and check if the two are equal?

(d) Find $(B \bigcup F)'$ and $B' \bigcap F'$ and check if the two are equal?

(e) Find $G \bigcap (C \bigcup F)$ and $(G \bigcap C) \bigcup (G \bigcap F)$ and check if the two are equal?

(f) Find $G \bigcup (C \bigcap F)$ and $(G \bigcup C) \bigcap (G \bigcup F)$ and check if the two are equal?

(g) Describe a partition and give two examples of partitions of the universal set A.

4. Toss two fair dice. Find the probability of the following event:

   (a) The sum of the two dice is 9.

   (b) The dice show the same number of dots.

   (c) The number shown by the second die is larger than the one shown by the first.

5. If the probability is 0.54 that Stock A will increase in value during the next month and the probability is 0.68 that Stock B will increase in value during the next month, what is the greatest possible value for the probability that neither of these two events will occur?

6. Questions on combination and permutation rules.

   (a) Five different books are on a shelf. In how many different ways could you arrange them?

   (b) How many different arrangements are there of the letters of the word "numbers"?

   (c) You have 5 shirts, but you will take with you only 3 for your vacation. In how many different ways can you do this?

   (d) Given 6 letters {a, b, c, d, e, f, g}, how many different 4-letter words can be formed from these 6 letters?

7. The Republican governor of a state is appointing a committee of five members to consider changes in the incom tax law. There are 15 state representatives – seven Democrats and eight Republicans – available for appointment to the committee. Assume that the governor selects the committee of five members randomly from the 15 representatives.

   (a) In how many different ways can the committee members be selected?

   (b) What is the probability that no Democrat is appointed to the committee?

(c) What is the probability that the majority of the committee members are Republican?

8. Republicans have two potential candidates (A, B) for presidential election, while the Democrats have three (1, 2, 3). Each party must select one to represent the party. The probability of Republicans selecting A is 0.8. If the Republicans choose A, then the probability of the Democrats choosing 1 is 0.5, choosing 2 is 0.3, and choosing 3 is 0.2. If the Republicans choose B, then the probability of the Democrats choosing 1 is 0.3, choosing 2 is 0.6, and choosing 3 is 0.1.

   (a) State the Bayes' rule.

   (b) Draw a tree diagram, and calculate all the intersection probabilities, that is, the probability each possible pair, e.g. $P(A \cap 1), P(A \cap 2)$, etc.

   (c) Find the conditional probability of the Republicans choosing A, given that the Democrats choose 2.